

Observation on the Bi-quadratic Equation with Five Unknowns

$$(x-y)(x^3 + y^3) = (1+12k^2)(X^2 - Y^2)w^2$$

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Abstract: In this paper we find substantially many non-zero numeral quintuples p,q,d,e,t fulfilling the bi-quadratic equation with five unknowns $(x-y)(x^3 + y^3) = (1+12k^2)(X^2 - Y^2)w^2$ different attractive dealings among the solutions and out of the ordinary numbers, octahedral numbers, centered multilateral & pyramidal numbers are exhibited.

Key words: Bi-Quadratic equation with five unknowns, essential solutions, multilateral number, Pyramidal numbers, Centered multilateral.

2010 Mathematics subject classification: 11D25.

Notations Used:

- $T_{m,n}$ - Polygonal number of rank n with size m.
- P_m^n - Pyramidal number of rank n with size m.
- Pr_n - Pronic number of rank n.
- OH_n - Octahedral number of rank n.

1. INTRODUCTION

Bi-quadratic Diophantine Equations, uniform and ununiform, have aroused the attention of many Mathematicians since uncertainty as can be seen from [1,2,17-19]. In the background one may refer [3-16] for varieties of problems on the Diophantine equations with two, three and four variables. This communiqué concerns with the problems of determining non-zero fundamental solutions of bi-quadratic equation in the six unknowns represented by

$(x-y)(x^3 + y^3) = (1+12k^2)(X^2 - Y^2)w^2$.
A small number of attractive relations between the

$$x = u + 2v, y = u - 2v, X = 2u + v, Y = 2u - v$$

solutions and particular multilateral numbers are offered.

2. METHOD OF ANALYSIS

The Diophantine equation representing the bi-quadratic equation with six unknowns below deliberation is

$$(x-y)(x^3 + y^3) = (1+12k^2)(X^2 - Y^2)w^2 \quad (1)$$

Introducing the linear transformations

(2)

in (1), it simplifies to

$$u^2 + 12v^2 = (1 + 12k^2)w^2 \quad (3)$$

The above equation (3) is solved through diverse methods and thus one obtains different sets of integer solutions to equation (1).

Set.1

$$\text{Let } w = a^2 + 12b^2 \quad (4)$$

Substituting (4) in (3) and using method of factorization, define

$$(u + i 2\sqrt{3}v) = (1 + i 2\sqrt{3}k)(a + i 2\sqrt{3}b)^2 \quad (5)$$

Equating real and imaginary parts, we have

$$\begin{aligned} u &= a^2 - 12b^2 - 24kab \\ v &= k(a^2 - 12b^2 + 2ab) \end{aligned} \quad (6)$$

Substituting the values of u and v in equation (2), the non-zero distinct integral solutions of (1) are given by

$$\begin{aligned} x &= x(a, b) = a^2 + 2ka^2 - 12b^2 - 24kb^2 + 4ab - 24kab \\ y &= y(a, b) = a^2 - 2ka^2 - 12b^2 + 24kb^2 - 4ab - 24kab \\ X &= X(a, b) = 2a^2 + ka^2 - 24b^2 - 12kb^2 + 2ab - 48kab \\ Y &= Y(a, b) = 2a^2 - ka^2 - 24b^2 + 12kb^2 - 2ab - 48kab \\ w &= w(a, b) = a^2 + 12b^2 \end{aligned} \quad (7)$$

Properties:

- i) $x(a, a+1) + y(a, a+1) + 2(11t_{4,a} + 48kt_{3,a}) \equiv -24 \pmod{48}$
- ii) $x(b+1, b) - y(b+1, b) + 4k(11t_{4,b} - 2b+1) - 8Pr_b \equiv 0$
- iii) $x(a, 2a^2 + 1) + y(a, 2a^2 + 1) + 144k OH_a + 2(48t_{4,a^2} + 47t_{4,a} + 12) = 0$
- iv) $x(2b^2 - 1, b) + w(2b^2 - 1, b) - 2(4t_{4,b^2} - 4t_{4,b} + 2SO_b + 1) - 2k(4t_{4,b^2} - 16t_{4,b} - 12SO_b + 1) = 0$
- v) $x(a, 2a - 1) - w(a, 2a - 1) + 4(24t_{4,a} - t_{6,a}) + 2k(47t_{4,a} - 12t_{6,a} - 48a + 12) \equiv -24 \pmod{96}$
- vi) $X(a, 3a - 1) + Y(a, 3a - 1) + 428t_{4,a} + 192kt_{5,a} \equiv -48 \pmod{288}$
- vii) $X(4b - 3, b) - Y(4b - 3, b) - 4t_{10,b} - 2k(4t_{4,b} - 24b + 9) \equiv 0$

Note:

In (5) replace $(1 + i 2\sqrt{3}k)$ by $(-1 + i 2\sqrt{3}k)$

$$(u + i 2\sqrt{3}v) = (-1 + i 2\sqrt{3}k)(a + i 2\sqrt{3}b)^2 \quad (8)$$

Following the process offered in set. 1 a diverse solution is given by

$$\begin{aligned} x &= x(a, b) = -a^2 + 2ka^2 - 12b^2 - 24kb^2 - 4ab - 24kab \\ y &= y(a, b) = -a^2 - 2ka^2 + 12b^2 + 24kb^2 + 4ab - 24kab \\ X &= X(a, b) = -2a^2 + ka^2 + 24b^2 - 12kb^2 - 2ab - 48kab \\ Y &= Y(a, b) = -2a^2 - ka^2 + 24b^2 + 12kb^2 + 2ab - 48kab \\ w &= w(a, b) = a^2 + 12b^2 \end{aligned} \quad (9)$$

Set .2

(3) can be written as

$$u^2 + 12v^2 = (1 + 12k^2)w^2 * 1 \quad (10)$$

Write 1 as $1 = \frac{(2 + i 2\sqrt{3})(2 - i 2\sqrt{3})}{16}$ (11)

Using (11) in (10) and employing the method of factorization, delineate

$$u + i 2\sqrt{3}v = (1 + i 2\sqrt{3}k)(a + i 2\sqrt{3}b)^2 * \left(\frac{2 + i 2\sqrt{3}}{4} \right) \quad (12)$$

Equating real and imaginary parts & replacing a by 4a and b by 4b, we have

$$\begin{aligned} u &= 8a^2 - 48a^2k - 96b^2 + 576b^2k - 96ab - 192abk \\ v &= 4a^2 + 8a^2k - 48b^2 - 96b^2k + 16ab - 96abk \end{aligned} \quad (13)$$

Using (13) & (2) we get the integral solutions of (1) to be

$$\begin{aligned} x &= x(a, b) = 16a^2 - 32ka^2 - 192b^2 + 384kb^2 - 64ab - 384kab \\ y &= y(a, b) = -64ka^2 + 768kb^2 - 128ab \\ X &= X(a, b) = 20a^2 - 88ka^2 - 240b^2 + 1056kb^2 - 176ab - 480kab \\ Y &= Y(a, b) = 12a^2 - 104ka^2 - 144b^2 + 1248kb^2 - 208ab - 288kab \\ w &= w(a, b) = 16a^2 + 192b^2 \end{aligned} \quad (14)$$

Properties:

- i) $x(a, 2a^2 - 1) + y(a, 2a^2 - 1) - 16(49t_{4,a} - 48t_{4,a^2} - 12SO_a - 12) + 32k(147t_{4,a} + 144t_{4,a^2} - 12SO_a + 36) = 0$
- ii) $2x(b+1, b) - y(b+1, b) + 352t_{4,b} + 768k \text{Pr}_b \equiv 1 \pmod{64}$

$$iii) x(a, 2a^2 + 1) + w(a, 2a^2 + 1) - 32(t_{4,a} - 6OH_a) - 32k(48t_{4,a^2} - 47t_{4,a} + 36OH_a) = 0$$

$$iv) x(b+1, b) - w(b+1, b) + 32k(12Obl_b - 11t_{4,b} + 2b + 1) + 64(Obl_b - 6t_{4,b}) = 0$$

$$v) y(a^2, a+1) + w(a^2, a+1) + 16(16P_a^5 + t_{4,a^2} + 12t_{4,a}) + 64k(t_{4,a^2} + 12t_{4,a} + 24a + 12) \equiv -192 \pmod{384}$$

$$vi) y(a, 2a^2 - 1) - w(a, 2a^2 - 1) + (64k + 16)(48t_{4,a^2} - 47t_{4,a} + 12) = 0$$

$$vii) X(1, 2b - 1) - Y(1, 2b - 1) + 16k(48t_{4,b} + 2GnO_b + 47b + 12) + 32(12t_{4,b} + 1) \equiv -88 \pmod{384}$$

$$viii) X(a, 7a - 5) + Y(a, 7a - 5) + 32(587t_{4,a} + 24t_{9,a}) + 64k(1761t_{4,a} - 24t_{9,a} - 2520a + 900) \equiv -9600 \pmod{26880}$$

Note :

In (12) replace $\left(\frac{2+i2\sqrt{3}}{4}\right)$ by $\left(\frac{-2+i2\sqrt{3}}{4}\right)$

$$u + i2\sqrt{3}v = (1 + i2\sqrt{3}k)(a + i2\sqrt{3}b)^2 * \left(\frac{-2+i2\sqrt{3}}{4}\right) \quad (15)$$

Following the process offered in set.2 a diverse solution is given by

$$\begin{aligned} x &= x(a, b) = -64ka^2 + 768kb^2 - 128ab \\ y &= y(a, b) = -16a^2 - 32ka^2 + 192b^2 + 384kb^2 - 64ab + 384kab \\ X &= X(a, b) = -12a^2 - 104ka^2 + 144b^2 + 1248kb^2 - 208ab + 288kab \\ Y &= Y(a, b) = -20a^2 - 88ka^2 + 240b^2 + 1056kb^2 - 176ab + 480kab \\ w &= w(a, b) = 16a^2 + 192b^2 \end{aligned} \quad (16)$$

Case 1:

$$u^2 + 12v^2 = (1 + 12k^2)w^2 * 1$$

Instead of (11), write 1 as

$$1 = \left(\frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{49}\right)xc \quad (17)$$

Using (17) in (10) and employing the method of factorization, define

$$u + i2\sqrt{3}v = (1 + i2\sqrt{3}k)(a + i2\sqrt{3}b)^2 * \left(\frac{1+i4\sqrt{3}}{7}\right) \quad (18)$$

Equating real and imaginary parts and replacing a by 7a and b by 7b

$$\begin{aligned} u &= 7a^2 - 168ka^2 - 84b^2 + 2016kb^2 - 336ab - 168kab \\ v &= 14a^2 + 7ka^2 - 168b^2 - 84kb^2 + 14ab + 336kab \end{aligned} \quad (19)$$

Using (19) & (2) we get the integral solutions of (1) to be

$$\begin{aligned} x &= x(a,b) = 35a^2 - 322ka^2 - 420b^2 + 1948kb^2 - 308ab + 504kab \\ y &= y(a,b) = -21a^2 - 182ka^2 + 252b^2 + 2184kb^2 - 364ab - 840kab \\ X &= X(a,b) = 28a^2 - 329ka^2 - 336b^2 + 3948kb^2 - 688ab \\ Y &= Y(a,b) = -343ka^2 + 4116kb^2 - 686ab - 672kab \\ w &= w(a,b) = 49a^2 + 588b^2 \end{aligned} \quad (20)$$

Note

In (18) replace $\left(\frac{1+i\sqrt{3}}{7}\right)$ by $\left(\frac{-1+i\sqrt{3}}{7}\right)$

$$u + i2\sqrt{3}v = (1+i2\sqrt{3}k)(a+i2\sqrt{3}b)^2 * \left(\frac{-1+i\sqrt{3}}{7}\right) \quad (21)$$

Following the procedure presented in case 1,a different solution is given by

$$\begin{aligned} x &= x(a,b) = 21a^2 - 182ka^2 - 252b^2 + 2184kb^2 - 364ab - 504kab \\ y &= y(a,b) = -35a^2 - 154ka^2 + 420b^2 + 1848kb^2 - 308ab + 840kab \\ X &= X(a,b) = -343ka^2 + 4116kb^2 - 686ab \\ Y &= Y(a,b) = -28a^2 - 329ka^2 + 336b^2 + 3948kb^2 - 658ab + 672kab \\ w &= w(a,b) = 49a^2 + 588b^2 \end{aligned} \quad (22)$$

Case 2:

$$u^2 + 12v^2 = (1+12k^2)w^2 * 1$$

As an alternative of (17), write 1 as

$$1 = \left(\frac{(8+i8\sqrt{3})(8-i8\sqrt{3})}{256}\right) \quad (23)$$

Using (17) in (10) and employing the method of factorization, define

$$u + i2\sqrt{3}v = (1+i2\sqrt{3}k)(a+i2\sqrt{3}b)^2 * \left(\frac{8+i8\sqrt{3}}{16}\right) \quad (24)$$

Equating real and imaginary parts and replacing a by 16a and b by 16b

$$\begin{aligned} u &= 128a^2 - 768ka^2 - 1536b^2 + 9216kb^2 - 1536ab - 3072kab \\ v &= 64a^2 + 128ka^2 - 768b^2 - 1536kb^2 + 256ab - 1536kab \end{aligned} \quad (25)$$

Using (25) & (2) we get the integral solutions of (1) to be

$$\begin{aligned} x &= x(a,b) = 256a^2 - 512ka^2 - 3072b^2 + 6144kb^2 - 1024ab - 6144kab \\ y &= y(a,b) = -1024ka^2 + 12288kb^2 - 2048ab \\ X &= X(a,b) = 320a^2 - 1408ka^2 - 3840b^2 + 16896kb^2 - 2816ab - 7680kab \\ Y &= Y(a,b) = 192a^2 - 1664ka^2 - 2034b^2 + 19968kb^2 - 3328ab - 4608kab \\ w &= w(a,b) = 256a^2 + 3072b^2 \end{aligned} \quad (26)$$

Note

In (24) replace $\left(\frac{8+i\sqrt{3}}{16}\right)$ by $\left(\frac{-8+i\sqrt{3}}{16}\right)$

$$u + i\sqrt{3}v = (1 + i\sqrt{3}k)(a + i\sqrt{3}b)^2 * \left(\frac{-8+i\sqrt{3}}{16}\right) \quad (27)$$

Following the parallel process as in case 2, the equivalent numeral solutions of (1) are given by

$$\begin{aligned} x &= x(a,b) = -1024ka^2 + 12288kb^2 - 2048ab \\ y &= y(a,b) = -256a^2 - 512ka^2 + 3072b^2 + 6144kb^2 - 1024ab + 6144kab \\ X &= X(a,b) = -192a^2 - 1644ka^2 + 2034b^2 + 19968kb^2 - 3328ab + 4608kab \\ Y &= Y(a,b) = -320a^2 - 1408ka^2 + 3840b^2 + 16896kb^2 - 2816ab + 7680kab \\ w &= w(a,b) = 256a^2 + 3072b^2 \end{aligned} \quad (28)$$

Set.III

Rewrite (3), we get

$$u^2 - w^2 = 12(k^2w^2 - v^2) \quad (29)$$

Choice: I

$$\frac{u+w}{3(kw+v)} = \frac{4(kw-v)}{u-w} = \frac{\alpha}{\beta}, \quad \beta \neq 0 \quad (30)$$

Using method of cross ratio, we get

$$\begin{aligned} u(\alpha, \beta, k) &= -3\alpha^2 + 4\beta^2 - 24k\alpha\beta \\ v(\alpha, \beta, k) &= 3\alpha^2k - 4\beta^2k - 2\alpha\beta \\ w(\alpha, \beta, k) &= -3\alpha^2 - 4\beta^2 \end{aligned} \quad (31)$$

Hence in observation of (2) the parallel non-zero numeral solutions of (1) are

$$x = x(\alpha, \beta, k) = -3\alpha^2 + 6\alpha^2k + 4\beta^2 - 8\beta^2k - 4\alpha\beta - 24\alpha\beta k$$

$$y = y(\alpha, \beta, k) = -3\alpha^2 - 6\alpha^2k + 4\beta^2 + 8\beta^2k + 4\alpha\beta - 24\alpha\beta k$$

$$X = X(\alpha, \beta, k) = -6\alpha^2 + 3\alpha^2k + 8\beta^2 - 4\beta^2k - 2\alpha\beta - 48\alpha\beta k$$

$$Y = Y(\alpha, \beta, k) = -6\alpha^2 - 3\alpha^2k + 8\beta^2 + 4\beta^2k + 2\alpha\beta - 48\alpha\beta k$$

$$w = w(\alpha, \beta, k) = -3\alpha^2 - 4\beta^2$$

Properties

$$i) x(\alpha, 2\alpha^2 + 1, k) + y(\alpha, 2\alpha^2 + 1, k) - 2(16t_{4,\alpha^2} + 13t_{4,\alpha} - 72kOH_\alpha + 4) = 0$$

$$ii) x(\alpha, 2\alpha^2 - 1, k) + y(\alpha, 2\alpha^2 - 1, k) + 2k(32t_{4,\alpha^2} + 38t_{4,\alpha} - 8) + 8SO_\alpha = 0$$

$$iii) x(\alpha, \alpha + 1, k) + w(\alpha, \alpha + 1, k) + 2(3t_{4,\alpha} + 2Pr_\alpha) + 2k(t_{4,\alpha} + 12Pr_\alpha + 8\alpha + 4) = 0$$

$$iv) x(\beta, 2\beta^2 - 1, k) + y(\beta, 2\beta^2 - 1, k) - 4(8t_{4,\beta^2} - 8t_{4,\beta} - SO_\beta + 2) - 2k(-13t_{4,\beta^2} + 16t_{4,\beta} - S12O_\beta - 4) = 0$$

$$v) y(\alpha, 4\alpha - 3, k) + w(\alpha, 4\alpha - 3, k) - 2(29t_{4,\alpha} + 2t_{10,\alpha}) - 2k(61t_{4,\alpha} - 12t_{10,\alpha} - 96\alpha + 36) \equiv 0 \pmod{96}$$

$$vi) y(5\beta - 3, \beta, k) - w(5\beta - 3, \beta, k) + 8(t_{7,\beta} - t_{4,\beta}) + 2k(24t_{7,\beta} + 79t_{4,\beta} - 90\beta + 27) = 0$$

$$vii) X(\alpha, 3\alpha - 2, k) + Y(\alpha, 3\alpha - 2, k) - 84t_{4,\alpha} + 96kt_{8,\alpha} \equiv 64 \pmod{192}$$

$$viii) X(\beta + 1, \beta, k) - Y(\beta + 1, \beta, k) + 2k(t_{4,\beta} - 6\beta - 3) + 4Pr_\beta = 0$$

In addition to (30), (29) may also be uttered in the form of ratios in three diverse choices that are obtainable below

Choice: II

$$\frac{u+w}{4(kw+v)} = \frac{3(kw-v)}{u-w} = \frac{\alpha}{\beta}$$

Choice: III

$$\frac{u+w}{2(kw+v)} = \frac{6(kw-v)}{u-w} = \frac{\alpha}{\beta}$$

Choice: IV

$$\frac{u+w}{2(kw-v)} = \frac{3(kw+v)}{u-w} = \frac{\alpha}{\beta}$$

Solving each of the exceeding system of equation by the subsequent process as offered in set.III, the parallel integer solution to (1) are establish to be as given below

Solution for choice II:

$$\begin{aligned}x &= x(\alpha, \beta, k) = -4\alpha^2 + 8\alpha^2k + 3\beta^2 - 6\beta^2k - 4\alpha\beta - 24\alpha\beta k \\y &= y(\alpha, \beta, k) = -4\alpha^2 - 8\alpha^2k + 3\beta^2 + 6\beta^2k + 4\alpha\beta - 24\alpha\beta k \\X &= X(\alpha, \beta, k) = -8\alpha^2 + 4\alpha^2k + 6\beta^2 - 3\beta^2k - 2\alpha\beta - 48\alpha\beta k\end{aligned}$$

$$\begin{aligned}Y &= Y(\alpha, \beta, k) = -8\alpha^2 - 4\alpha^2k + 6\beta^2 + 3\beta^2k + 2\alpha\beta - 48\alpha\beta k \\w &= w(\alpha, \beta, k) = -4\alpha^2 - 3\beta^2\end{aligned}$$

Solution for choice III:

$$\begin{aligned}x &= x(\alpha, \beta, k) = -2\alpha^2 + 4\alpha^2k + 6\beta^2 - 12\beta^2k - 4\alpha\beta - 24\alpha\beta k \\y &= y(\alpha, \beta, k) = -2\alpha^2 - 4\alpha^2k + 6\beta^2 + 12\beta^2k + 4\alpha\beta - 24\alpha\beta k \\X &= X(\alpha, \beta, k) = -4\alpha^2 + 2\alpha^2k + 12\beta^2 - 6\beta^2k - 2\alpha\beta - 48\alpha\beta k \\Y &= Y(\alpha, \beta, k) = -4\alpha^2 - 2\alpha^2k + 12\beta^2 + 6\beta^2k + 2\alpha\beta - 48\alpha\beta k \\w &= w(\alpha, \beta, k) = -2\alpha^2 - 6\beta^2\end{aligned}$$

Solution for choice IV:

$$\begin{aligned}x^3 + y^3 &= (x - y)^4 x, \text{ reflections des ERA-} \\x &= x(\alpha, \beta, k) = 2\alpha^2 + 4\alpha^2k - 6\beta^2 - 12\beta^2k - 4\alpha\beta + 24\alpha\beta k \quad [4] \text{ Gopalan .M.A and Janaki.G., (2009),} \\y &= y(\alpha, \beta, k) = 2\alpha^2 - 4\alpha^2k - 6\beta^2 + 12\beta^2k + 4\alpha\beta + 24\alpha\beta k \quad \text{Observation on } (x^2 - y^2)4xy = z^4 x, \text{ Acta} \\X &= X(\alpha, \beta, k) = 4\alpha^2 + 2\alpha^2k - 12\beta^2 - 6\beta^2k - 2\alpha\beta + 48\alpha\beta k \quad \text{ciencia Indica, Vol.XXXVIII, No.2,445} \\Y &= Y(\alpha, \beta, k) = 4\alpha^2 - 2\alpha^2k - 12\beta^2 + 6\beta^2k + 2\alpha\beta \quad [5] \text{ Gopalan .M.A., Vidhyalakshmi.S and} \\w &= w(\alpha, \beta, k) = 2\alpha^2 + 6\beta^2 \quad \text{Devibala.S, (2010), Ternary Quartic} \\&\quad \text{Diophantine Equation} \\&\quad 2^{4n+3}(x^3 - y^3) = z^4 x,\end{aligned}$$

[6] Gopalan M.A., Vijayasankar.A and Manju Somnath, (2010) Integral solutions of $x^2 - y^2 = z^4 x$, Impact journal of Science and Technology, Vol.2(4), No. 149-157.

[7] Gopalan .M.A.,and Shanmuganandham.P., (2010), On the Biquadratic equation $x^4 + y^4 + z^4 = 2w^4 x$, Impact journal of Science and Technology, Vol.4, No4, 111-115.

[8] Gopalan M.A., Sangeetha.G.,(2011), Integral solutions of ternary non-homogeneous biquadratic equation $x^4 + x^2 + y^2 - y = z^2 + z$, Acta ciencia indica, Vol.XXXVII M, No.4,799-803.

[9] Gopalan .M.A..Vidhyalakshmi.S., Sumathi.G,(2012), On the ternary biquadratic non-homogeneous equation $(2k+1)(x^2 + y^2 + xy) = z^4$, Indian journal of Engineering,Voll, No.1.

[10] Gopalan .M.A..Vidhyalakshmi.S., Sumathi.G,(2012), Integral solutions of ternary biquadratic non-homogeneous equation

3. CONCLUSION:

In this paper, we have offered distinct choices of integral solutions to homogeneous biquadratic equation with six unknowns,

$(x - y)(x^3 + y^3) = (1 + 12k^2)(X^2 - Y^2)w^2$
To terminate, as biquadratic equations are rich in multiplicity, one may regard as other forms of biquadratic equations and rummage around for analogous properties.

REFERENCES

- [1] Carmicheal R.D, (1959), The theory of numbers and Diophantine analysis, Dover Publications, New York.
- [2] Dickson I.E., (1952), History of theory of numbers, Vol.2, Chelsia Publishing Co., New York.
- [3] Gopalan .M.A., and Anbuselvi.R., (2009), Integral solutions of binary quartic equation

$$(\alpha + 1)(x^2 + y^2) + (2\alpha + 1)xy = z^4 \quad ,$$

JRACE, Vol.6, No.2, 97-98.

- [11] Gopalan .M.A..Vidhyalakshmi.S., Sumathi.G,(2013), Integer solutions of ternary biquadratic non-homogeneous equation $(k+1)(x^2 + y^2) - (2k+1)xy = z^4 \quad ,$ Archimeds J.Math,3(1),67-71.
- [12] Gopalan .M.A.,Geetha.V,(2013),Integral solurtions of ternary biquadratic equation $x^2 + 13y^2 = z^4$, IJLRST,Vol.2, Issue 2, 59-61.
- [13] Gopalan .M.A..Vidhyalakshmi.S., Sumathi.G,(2013), On the ternary biquadratic non-homogeneous equation $x^2 + xy^3 = z^4$,
- [14] Gopalan .M.A., Vidhyalakshmi.S., Kavitha.A., (2013),Integral points on the biquadratic equation $(x + y + z)^3 = z^2(3xy - x^2 - y^2)$,IJMSEA,Vol.7, No.1,81-84.
- [15] Gopalan .M.A., Vidhyalakshmi. S., Mallika.S,(2013),Integral solutions of $2(x^2 + y^2) + 3xy = (\alpha^2 + 7)^n z^4$, IJMIE, Vol.3, No.5, 408-414.
- [16] Gopalan M.A and Sivakami. B,(2013),Integral solutions of quadratic equation with four unknowns $x^3 + y^3 + z^3 = 3xyz + 2(x + y)w^3 \quad ,$ Antarctica J.Math., 10(2), 151-159.
- [17] Mordell L.J, (1970) Diophantine Equations, Academic Press, New York,
- [18] Nigel,Smart.P,(1999), The Algorithmic Resolutoins of Diophantine Equations, Cambridge University Press,london
- [19] Telang S.G., (1996), Number Theory,Tata Mcgraw Hill Publishing company, New Delhi